ACCOUNTING FOR GROWTH: THE ROLE OF PHYSICAL WORK

Robert U Ayres & Benjamin Warr
Center for the Management of Environmental Resources
INSEAD
Fontainebleau, France
Email: robert.ayres@insead.edu

Abstract

This paper tests several related hypothesis for explaining US economic growth since 1900. It begins from the belief that consumption of natural resources – especially energy (or, more precisely, exergy) — has been, and still is, an important factor of production and driver of economic growth. However the major result of the paper is that it is not `raw’ energy (exergy) as an input, but exergy converted to useful (physical) work that – along with capital and (human) labor – really explains output and drives long-term economic growth. We develop a formal model (Resource-EXergy Service or REXS) based on these ideas. Using this model we demonstrate first that, if raw energy inputs are included with capital and labor in a Cobb-Douglas or any other production function satisfying the Euler (constant returns) condition, the 100-year growth history of the US cannot be explained without introducing an exogenous ‘technical progress’ multiplier (the Solow residual) to explain most of the growth. However, if we replace raw energy as an input by `useful work’ (the sum total of all types of physical work by animals, prime movers and heat transfer systems) as a factor of production, the historical growth path of the US is reproduced with high accuracy from 1900 until the mid 1970s, without any residual except during brief periods of economic dislocation, and with fairly high accuracy since then. (There are indications that an additional factor, possibly information technology, needs to be taken into account as a fourth input factor since the 1970s.) Various hypotheses for explaining the latest period are discussed briefly, along with future implications.
1. Introduction

The primary motivation of this paper is to revisit the neoclassical theory of growth from the physical (thermodynamic) perspective. The ‘standard’ growth theory, which was formulated in its current production function form (independently) by Robert Solow and Trevor Swan [Solow 1956; Swan 1956; Solow 1957]. The standard theory assumes that production of goods and services (in monetary terms) can be expressed as a function of capital and labor, yet the major contribution to growth had to be attributed to an unexplained exogenous driver called ‘technological progress’.

Both casual observation and physical intuition have convinced many investigators since Georgescu-Roegen first expounded on the subject, that production in the real world cannot be understood without taking into account the role of materials and energy [Georgescu-Roegen 1971]. Our primary objective in this paper is to elaborate and quantify this intuition – which we share – and to simultaneously endogenize ‘technological progress’, insofar as possible. A further, though secondary, objective is to clarify the differences between our current approach and the several earlier attempts to incorporate resource flows explicitly into growth models [Jorgenson and Houthakker 1973; Allen et al 1976; Hannon and Joyce 1981; Jorgenson 1983; Jorgenson 1984]. We attempt to explain, hereafter, why the several earlier attempts did not succeed and how – and why – the present approach differs from earlier ones.

Before passing on, we also emphasize that several features of our work follow (albeit indirectly) from our concept of growth dynamics as a positive feedback cycle. This may not seem immediately relevant to our main results. But it is relevant to some of the choices we make later in formalizing the growth model. The generic positive feedback cycle, in economics, operates as follows: cheaper resource inputs, due to discoveries, economies of scale and experience (or learning-by-doing) enable tangible goods and intangible services to be produced and delivered at ever lower cost. This is another way of saying that resource flows are productive, which is our point of departure. Lower cost, in competitive markets, translates into lower prices for all products and services. Thanks to non-zero price elasticity, lower prices encourage higher demand. Since demand for final goods and services necessarily corresponds to the sum of factor payments, most of which go back to labor as wages and salaries, it follows that wages of labor tend to increase as output rises. This, in turn, stimulates the further substitution of natural resources, especially fossil fuels, and mechanical power produced from resource inputs, for human (and animal) labor. This continuing substitution drives further increases in scale, experience, learning and still lower costs.

Based on both qualitative and quantitative evidence, the existence of the positive feedback cycle sketched briefly above implies that physical resource flows must be a major factor of production. Indeed, including a fossil energy flow proxy in the neoclassical production function, without any constraint on factor share, seems to account for economic growth quite accurately, at least for limited time periods, without any exogenous time-dependent term [Hannon and Joyce 1981; Kümmel 1982b; Cleveland et al 1984; Kümmel et al 1985; Kümmel 1989; Kaufmann 1992; Beaudreau 1998; Cleveland et al 1998; Kümmel et al 2000]. It is important to note, however, that including energy or exergy as a factor of production does not explain economic growth for periods longer than two or three decades, without recalibration or
without a time dependent multiplier. The reason for this (negative) empirical result becomes clear hereafter.

The fact that economic growth tends to be very closely correlated with energy consumption, at least for short periods does not \textit{a priori} mean that energy consumption is the cause of the growth. Indeed, many economic growth models still assume exactly the opposite: that economic growth (due to accumulation of capital, and labor, plus technical progress) is responsible for increasing energy and natural resource consumption. This automatically explains (indeed, guarantees) high correlation. We argue, on the contrary, that declining resource prices can have a direct impact on growth, via the positive feedback loop. The direction of causality must evidently be determined empirically by other means, either theory based or empirical.

The major new feature of our approach is that, in contrast to earlier treatments that introduced (commercial) energy(exergy), or energy(exergy) and materials separately, as factors of production, we consider \textit{physical work} (or `exergy services') as the appropriate independent variable for the production function. The term exergy is introduced and explained in section 2 which follows. It is important to emphasize here that \textit{physical work} is a well-defined concept from thermodynamics and physics; it must be distinguished from the term as it is used in ordinary language, where `work' is generally what people do to earn a living. The relationship between potential work (exergy) and actual work – or exergy services – performed in the economy is explained in \textit{Section 3}. In brief, the ratio of actual work to potential work can be interpreted as the \textit{thermodynamic efficiency} with which the economy converts resource inputs into finished materials and services.

To avoid confusion, it is important to note that term `thermodynamic efficiency', introduced above, is not related to economic efficiency. Thermodynamic efficiency is a straightforward ratio between (physical work) output and resource input. As will be seen, both numerator and denominator are measured in the same physical units (e.g. gigajoules or GJ). Moreover, we are able to estimate both inputs and outputs, and the resulting ratio, with reasonable accuracy, from published empirical data. (See \textit{Sections 2 and 3}).

Introducing an additional factor creates certain conceptual problems that we must acknowledge from the outset. Suppose we had opted (as some modelers have) to choose exergy inputs as a factor of production, measured in monetary terms. Payments for fossil fuels, minerals, ores, farm products and other forms of `raw' exergy inputs are actually payments for `produced' outputs of the extractive industries, agriculture and forest products sectors. By convention, all of these are intermediates, accounting for a very small percentage of GDP, – perhaps 4% without agriculture and less than 10% even if agriculture is included. Evidently, electric power, motive power, space heat and industrial heat are also produced outputs. Of course, some capital and labor are required to produce these intermediate products.

However, among these exergy services only electric power is regarded as a \textit{commodity} produced and sold by a well-defined industrial sector for which financial accounts are kept. Motive power is produced and consumed (mostly) within the agriculture, transportation and construction sectors, while heat is produced and consumed within many other sectors, including households. They are not regarded as (or, accounted for) commodities, and they do not have explicit market prices. If shadow prices for these kinds of exergy services (useful work) were available, it is likely that the corresponding payments would account – \textit{in toto} – for a considerably greater share of the US GDP. But, needless to say, capital and labor, as well as
inputs from the extractive and farming sectors, are also required to produce these intermediates, just as they, in turn, are required to produce other goods and services.

In short, to introduce either ‘raw’exergy or exergy services as a third factor of production also forces us to think in terms of a multi-sector input-output structure with inter-industry feedbacks. The two choices (exergy or exergy services) differ only in the magnitudes of the feedbacks from downstream products and services back to extraction and primary processing. At first glance this might argue against introducing either of them as a third factor.

Note that capital goods are also produced intermediates. The inputs to capital goods production are – again – capital, labor and other intermediates (including exergy and/or exergy services.) The key conceptual difference is that capital goods and labor are not consumed in the production process (although depreciation is almost a form of consumption), whence they are cumulable, and capital and labor services are proportional to the corresponding stocks. On the other hand, resource (exergy) flows, or exergy service flows, are not cumulable; they are consumed immediately in the production process.

Furthermore, thanks to cumulability, capital services and labor services can be – within limits – regarded as independent variables in the sense of being independent of current economic conditions (i.e. demand vis a vis potential supply). Of course, the true relationship between capital and output is one of mutual dependence, but with a time lag between the output level and the stock levels. It takes a few years for capital stocks to respond to current economic conditions via the price mechanism. The potential labor supply responds through demographic feedbacks over an even longer time frame, whence adjustment of current labor supply occurs mainly through the political process (i.e. laws regarding minimum schooling requirements, retirement ages, work-weeks, immigration, and so on). On the other hand, both resource (exergy) consumption, and exergy service (useful work) consumption levels respond rather quickly to economic conditions (via prices), whereas the forward impact of changes in prices on demand – up or down – driven by technological improvements and/or resource scarcity lags by several years.

Having acknowledged these points, the question arises: do they, taken together, preclude the use of exergy flows or exergy service flows as inputs to a formal production function? We think that the answer is ‘no’. We argue (Section 4) that the economic system should be understood as a sequential materials processing system, converting raw materials (and fuels) by stages into final products and services. The existence of (possibly lagged) feedbacks from downstream sectors to upstream sectors is understood. Capital services constitute one such lagged feedback. Exergy services can be regarded as a generic intermediate with both feedback and feed-forward. Whether it has explanatory power is then an empirical question.

Section 5 presents the formal Resource-EXergy Service (REXS) model, which is mainly defined by a choice of variables and production function. Section 6 presents the main results and Section 7 summarizes and discusses further implications.
2. The role of natural resources and energy (exergy)

An obvious implication of economic history – and one that is consistent with our view of growth dynamics as a feedback cycle – is that important ‘engine of growth’ since the first industrial revolution has been the continuously declining real price of physical resources, especially energy (and power) delivered at a point of use. The tendency of virtually all raw material and fuel costs to decline over time (lumber was the main exception) has been thoroughly documented, especially by economists at Resources For the Future (RFF) [Barnett and Morse 1962; Potter and Christy 1968; Smith and Krutilla 1979]. The increasing availability of energy from fossil fuels, and power from steam engines and internal combustion engines (ICEs), has clearly played a fundamental role in past economic growth. Machines powered by fossil energy have gradually displaced animals, wind power, water power and human muscles and thus made human workers vastly more productive than they would otherwise have been. There is no dispute among economists on this point.

The term *energy* as used above, and in most discussions (including the economics literature) is actually technically incorrect. The reason is that *energy* is conserved in every activity or process and therefore cannot be `used up’ – as most common usages of the term imply. But energy is not necessarily available to do useful work. The standard textbook example is the heat energy in the ocean water, virtually none of which can be utilized for doing useful work. As was discovered nearly two centuries ago by the French engineer Sadi Carnot, heat can only be converted into useful work if there is a temperature *gradient*. Absolute temperature does not matter. It is the temperature difference between two reservoirs that determines the amount of work that can be extracted by a so-called heat engine. By the same token, it is the temperature difference between the sun and the earth that drives most natural processes on earth, including the weather and photosynthesis.

*Exergy* is the correct thermodynamic term for `available energy’ or `useful energy’, or energy capable of performing mechanical, chemical or thermal work. The distinction is theoretically important because energy is a conserved quantity: this is the famous first law of thermodynamics. Energy is not `used up’ in physical processes, it is merely degraded from available to less and less available forms. On the other hand, exergy is dissipated (used and destroyed) in all transformation processes. The measure of exergy destruction is the production of a thermodynamic quantity called *entropy* (second law of thermodynamics).

The formal definition of exergy is the maximum work that could theoretically be done by a system as it approaches thermodynamic equilibrium with its surroundings, reversibly. Thus exergy is effectively equivalent to *potential work*. There is an important distinction between *potential work* and *actual work done* by animals or machines. The *conversion efficiency* between exergy (potential work), as an input, and actual work done, as an output, is also an important concept in thermodynamics. The notion of thermodynamic efficiency plays a key role in this paper.

To summarize: the technical definition of exergy is the maximum work that a subsystem can do as it approaches thermodynamic equilibrium (reversibly) with its surroundings. Exergy is also measured in energy units, and exergy values are very nearly the same as *enthalpy* (heating values) for all ordinary fuels. So, effectively, it is what most people mean when they speak of `energy’. The major exception to this rule is that exergy is a measure that is applicable, and can
be estimated with acceptable accuracy, not only for traditional fuels but to all agricultural products and industrial materials, including minerals. This point is important because it enables us to construct an aggregate measure of all resource flows into the economic system, as well as an aggregate measure of all processed intermediate flows. We have tabulated and published exergy values per kg for most common materials and mixtures (such as ores) in [Ayres and Ayres 1999a]. See Appendix A of this paper for more details.

3. Physical work and thermodynamic conversion efficiency

As noted above, exergy is equivalent to maximum potential work. There are several kinds of work, including mechanical work, electrical work and chemical work. For non-engineers, mechanical work can be exemplified in a variety of ways, such as lifting a weight against gravity or compressing a fluid. The term horsepower was introduced in the context of horses pumping water from flooded 18th century British coal and tin mines. A more general definition of work is movement against a potential gradient (or resistance) of some sort. A heat engine is a mechanical device to perform work from heat (though not all work is performed by heat engines.)

With this in mind, we can subdivide work into three broad categories, as follows: work done by animal (or human) muscles, work done by heat engines or water or wind turbines and work done in other ways (e.g. thermal or chemical work). Mechanical work can be further subdivided into work done to generate electric power and work done to provide motive power (e.g. to drive motor vehicles.) The power sources in this case are so-called ‘prime movers’, including all kinds of internal and external combustion engines, from steam turbines to jet engines. So called ‘renewables, including hydraulic, nuclear, wind and solar power sources for electric power generation are conventionally included. However electric motors are not prime movers, because electricity is generated by some other prime mover, usually a steam or gas turbine. In fact, electricity can be defined (for purposes of this paper) as ‘pure’ work.

Chemical work is exemplified by the reduction of metal ores to obtain the pure metal, or indeed to drive any endothermic chemical synthesis process. (Ammonia synthesis is a good example.) Thermal work is exemplified by the transfer of heat from its point of origin (e.g. a furnace) to its point of use, via one or more heat exchangers and a carrier (such as steam, hot water or hot air.).

To measure the useful work $U$ done by the economy, in practice, it is helpful to classify fuels by use. The first category is muscle work, for which the fuel is food or feed. In the US, human muscle work was quantitatively insignificant by 1900 and can be neglected. Horses and mules, which accounted for most animal work on US farms and urban transport, have not changed significantly since then. Animal work was still significant up to the 1930s but mechanical and electrical work have since become far more important. The thermodynamic efficiency with which horses and mules convert feed energy to useful work is generally reckoned at about 4% (i.e. one unit of work requires 25 units of feed).

The second category is fuel used by prime movers to do mechanical work. This consists of fuel used by electric power generation equipment and fuel used by mobile power sources such as motor vehicles, aircraft and so on. As regards mobile power sources, we define thermodynamic efficiency in terms of useful work performed by the whole vehicle, against air resistance and rolling resistance of the wheels on the road, not just work done by the engine
itself. Thus the efficiency of an automobile is the ratio of work done by the vehicle to the total potential work (exergy content) of the fuel.

The third broad category is fuel used to generate heat as such, either for industry (process heat to do chemical work) or space heat and domestic uses such as washing and cooking. The efficiency, in this case, refers to the delivery system. Lighting can be thought of as a special case of heating.

![Figure 1: Energy (exergy) conversion efficiencies, USA 1900-1998](image)

Clearly, the efficiency of muscles as energy convertors has not changed during human history. But the conversion efficiency of heat engines, domestic and commercial heating systems and industrial thermal processes has increased significantly over the past 100 years. We have plotted these increasing conversion efficiencies, from 1900 to 1998 in Figure 1. (Detailed derivations of these curves involve extensive reviews of the engineering literature and technological history. Details, including data sources, can be found in another publication [Ayres and Warr 2003].

Electrical work output is measured directly in kilowatt-hours (kwh) generated. Data are published by the US Federal Power Commission and the US Department of Energy (see Appendix A). Other types of work must be estimated from fuel inputs, multiplied by conversion efficiencies, as shown in Figure 1, over time. Allocations of fossil fuel exergy inputs to the economy by type of work are shown in Figure 2. Electrification has been perhaps the single most important source of useful work for production of goods and services, and (as will be seen...
later) the most important single driver of economic growth during the twentieth century. The fuel exergy required to generate a kilowatt-hour of electric power has decreased by a factor of ten during the past century. This implies that the thermodynamic efficiency of conversion increased over that period by the same factor, as shown in Figure 2.

Electricity prices fell correspondingly, especially during the first half of the century. However, the consumption of electricity in the US has increased over the same period by a factor of more than 1300, as shown in Figure 3. (This exemplifies the positive feedback economic `growth engine’ discussed briefly in the introduction.)
4. Towards a new theory of production and growth

Before proceeding further, it is important to mention one of the key assumptions of the standard theory, as set forth by Robert Solow, namely that marginal factor productivity can safely be equated with factor share in the national accounts. This simplistic assumption is particularly convenient for models based on Cobb-Douglas production functions. It is built into virtually all textbook discussions of growth theory, since the implications for labor and capital (marginal) productivity are easily derived. Labor gets the lion’s share of payments in the US national accounts, around 70 percent. Capital (defined as interest, dividends, rents and royalties) gets all of the rest, because all payments are attributable to one category or the other, by definition. The figures vary slightly from year to year, but they have been relatively stable for the past century or more. It follows that marginal labor productivity should be around 0.7 and marginal capital productivity should be around 0.3 in a Cobb-Douglas framework.

Payments to extractive resource owners (excluding farms) are hidden in the capital accounts, and they constitute a very small proportion—perhaps 3–4 percent—of GDP. This implies that resource productivity must be correspondingly small in comparison with labor or capital productivity. This has been a major source of confusion and misdirected effort in the past.

We reject this simple assumption (along with most modern modelers) on the basis of two arguments. The first follows from our view of the growth process as a positive feedback cycle,
as discussed previously. This implies that resource (exergy) flows – or, more precisely, declining resource prices – are not simply a consequence of growth. They are also (and simultaneously) a cause of growth. This means that the marginal productivity of resource flows should not be quantitatively insignificant compared to the marginal productivities of other factors. Nor should it be constant over a long period of time. There is an apparent inconsistency between very small factor payments directly attributable to physical resources – especially fossil fuels – and the obvious importance of energy (exergy) as a factor of production.

The second argument, which is more rigorous, is based on the fact that the identification of marginal factor productivities with factor shares in the national accounts is based on an oversimplification of the neoclassical theory of optimal income allocation. If labor and capital are the only two factors of production, neoclassical theory implies that the productivity of a factor of production must be proportional to the share of that factor in the national income. This proposition is quite easy to prove in a hypothetical single sector economy consisting of a large number of producers manufacturing a single all-purpose good using only labor and capital services. (The textbook example is usually bread, produced by bakeries that produce bread from capital and labor, but without any inputs of flour or fuel [Mankiw 1997].)

The supposed link between factor payments and factor productivities gives the national accounts a direct and fundamental (but spurious) role in production theory. In reality, however, (as noted in the introduction) the economy produces final products from a chain of intermediates, not directly from raw materials or, still less, from labor and capital without material inputs. In the simple single sector model used to `prove' the relationship between factor productivity and factor payments, this crucial fact is neglected. Allowing for the omission of intermediates (by introducing even a two-sector or three-sector production process) the picture changes completely. In effect, downstream value-added stages act as productivity multipliers. This enables a factor receiving a very small share of the national income directly, to contribute a much larger effective share of the value of aggregate production, i.e. to be much more productive than its share of overall labor and capital would seem to imply if the simple theory of income allocation were applicable [Ayres 2001a].

Our rejection of the simplistic identification of marginal productivities with factor shares has two consequences. One is that we are free to depart from the Cobb-Douglas strait-jacket. The other is that we must determine the parameters of the chosen production function by means of statistical fitting procedures. These issues are discussed in the next section.

For clarity in further discussion, we use the conventional terminology $L$ for human labor, as indexed to man-hours employed, and $K$ for produced capital (a construct of accumulated investment less depreciation), as compiled and published by the Bureau of Economic Analysis in the US Department of Commerce. We use the symbol $E$ for the energy inputs to the economy, as traditionally defined and compiled by the US Department of Energy. This consists of the heat (actually, enthalpy) content of fossil fuels and fuelwood, plus the nuclear heat used as an input to nuclear electric power generation, and the energy of flowing water harnessed for purposes of hydro-electric power production, plus small contributions from wind and solar heat. This variable has been used many times in the economics literature.

We use the symbol $B$ for exergy inputs to the economy, which include the items above –all of which are potential (but not actually performed) work –plus the potential work embodied in non-fuel wood and agricultural products and non-fuel minerals, such as sulfide ores. In
practice, the mineral contribution to exergy is quite small (except in the metallurgical industry itself) and can be neglected without significant error. The major quantitative difference between $E$ and $B$ nowadays is that the latter is slightly larger and more inclusive. However, in the 19th century and the early years of the 20th century (and in many developing countries) the differences are significant.

We use the symbol $U$ for work actually performed in the economy for economic purposes. The components of performed work include animal work (by horses and mules), work done by prime movers (both electric power generated and motive power) and heat delivered to a point of use, whether industrial or residential. The human contribution to physical (muscle) work can be neglected in comparison to other inputs as a first approximation. We distinguish two variants of performed work, namely $U_E$ and $U_B$ where the second variant includes animal work, whereas the first variant does not. The distinction is necessary because animals consume feed produced by the agriculture sector, which is included in $B$ but not included in the conventional measure of energy inputs to the economy, $E$.

Given the assumed importance of resource (exergy) flows in the economy, one might postulate two simple linear relationships of the form:

\[ Y = f_B g_E E = g_B U_B \]  

or

\[ Y = f_B g_B B = g_B U_B \]  

where $Y$ is GDP, measured in dollars, $E$ is a measure of commercial energy (mainly fossil fuels), $B$ is a measure of all `raw' physical resource inputs (technically, exergy), including fuels, minerals and agricultural and forest products. Then $f$ is the thermodynamic efficiency defined earlier, namely the ratio of `useful work performed' $U$ done by the economy as a whole to `raw' exergy input. Then $g$ is the ratio of economic output in value terms to useful work performed. The variables $f, g$ and $U$ have implicit subscripts $B$ or $E$, which we neglect hereafter where the choice is obvious from context. Since work appears in both numerator and denominator, its definition depends on whether we choose $B$ or $E$. Note that equations (1a,b) are essentially definitions of the two new variables $f, g$. There is no theory or approximation involved at this stage, except for the implicit assumption of linearity.

There are two mathematical conditions that a production function must satisfy to be economically realistic. One is the condition of constant (or nearly constant) returns to scale. This implies that the function should be a first order homogeneous function of its variables (known as the Euler condition). The other requirement is that the logarithmic derivatives (marginal productivities) of the factor variables should be non-negative – at least on average – throughout the entire time period (1900-1998).
Figure 4: GDP (Y) & factors of production K, L, B & E: USA 1900-1998
Subject to these requirements, we note that the expressions (1a) or (1b) can be converted into a production function in either of two ways. The first possibility is to specify either $E$ or $B$ as a plausible factor of production (along with $K, L$). Then the product $fg$ with subscripts $E$ or $B$ can be approximated by some first order homogeneous function of the three factors: labor $L$, capital $K$ and $E$ or $B$. The second possibility is that useful work $U$ is a more plausible factor of production (instead of $E$ or $B$) and the function $g$ can be expressed approximately by some first order homogeneous function of $K, L$ and $U$, again with appropriate subscripts. We have tested these possibilities empirically, for several choices of production function, as noted hereafter.

To begin with, the traditional variables capital $K$, and labor $L$, as usually defined for purposes of economic analysis, are plotted in Figure 4 from 1900 to 1998, along with deflated GDP and a traditional Cobb-Douglas production function of $K, L, E$. It is important to note that GDP increases faster than any of the three contributory input factors. The need for a time-dependent factor representing technical progress (i.e. the Solow residual) is evident as seen in the figure. Replacing $E$ by $B$ (i.e. including biomass) does not affect that qualitative conclusion. Substituting a more complex form of production function, whether CES, trans-log or LINEX (introduced later) with the same variables does not make a significant difference in the need for an exogenous multiplier, although the unexplained residual can be reduced slightly. The problem is, simply, that US GDP since 1900 has increased faster than $K, L$ or either $E$ or $B$, and therefore faster than any homogeneous first order combination of those variables. Thus, from here on, we drop the possibility of using either $E$ or $B$ as a factor of production in a production function.

We now turn to the alternative possibility, namely to try useful work $U$ (exergy services) as a factor of production instead of $E$ or $B$. The analogy with capital services seems apposite. Effectively there are two definitions of useful work to be considered hereafter, namely

\begin{equation}
U_B = f_B B \tag{2a}
\end{equation}

or

\begin{equation}
U_E = f_E E \tag{2b}
\end{equation}
As mentioned earlier, the ratios $f$ are, effectively, composite overall thermodynamic exergy conversion efficiencies. The former takes into account animal work and agricultural products, including animal feed. The latter neglects animal work and agricultural production. These two aggregate efficiency trends are calculated using exergy input data from 1900 to 1998, as shown in Figure 5 multiplied by estimated thermodynamic conversion efficiency trends plotted by type of useful work, in Figure 1. The results are plotted in Figure 6. Evidently if the trends in $f_E$ and $f_B$ are fairly steadily upward throughout a long period (such as a century) it would seem reasonably safe to project these trend curves into the future for two or three decades. The trends in physical work performed ($U_B$) and work/GDP ratio are shown in Figure 7.
Figure 6: Overall thermodynamic efficiencies
5. Choosing a production function

Having decided to introduce the broadest definition of useful work (or exergy services) $U_B$ as a factor of production, in addition to the usual capital $K$ and labor $L$, the choice of functional forms remains. As noted already, the Cobb-Douglas form is attractive in the case of two factors $K, L$ because the marginal productivities of capital and labor of can be equated with factor shares in the national accounts. But adding a third factor that is not independent of the other two, invalidates this argument. At first glance, this is unfortunate, because it means the parameters of the production function – whether Cobb-Douglas, CES, trans-log or other – will have to be determined by statistical fitting methods, with all the associated difficulties.

On the other hand, giving up the restrictive Cobb-Douglas assumption of constant marginal productivities over long time periods has a compensating advantage: it means that a better fit may be possible than pure C-D would allow. (In fact, this hope is realized.) Actually, the form of production function we have used was originally derived by reversing the usual logic [Kümmel et al 1985]: Instead of choosing a mathematical production function and performing logarithmic differentiation, one can choose simple mathematical forms for the three marginal productivities, (based on plausible assumptions about asymptotic behavior), and perform three partial integrations instead. The resulting linear-exponential (LINEX) form is given below. We have merely substituted $U_B$ for $E$ in Kümmel’s function, yielding
where, \( a \) and \( b \) are parameters to be chosen econometrically and \( A \) is a multiplier. If economic output and growth are fully explained by the three variables, then the multiplier \( A \) should be independent of time. It can be verified without difficulty that the r.h.s. of (3) satisfies the Euler condition for constant returns-to-scale. It can also be shown that the requirement of non-negative marginal productivities can be met.

As a matter of fact, the LINEX function has another useful feature that is worthy of mention. Namely, it does not imply (as does the Cobb-Douglas function) that the three factors are all strict substitutes for each other in the sense that more of one factor implies less of the other, or conversely. On the contrary, it implies a more complex and more realistic substitution-complement relationship among the variables.

The three factor productivities are easily derived by differentiation as follows:

\[
\frac{\partial Y}{\partial K} = \frac{bL}{K} \tag{4a}
\]

\[
\frac{\partial Y}{\partial L} = \frac{aL}{U} - \frac{bL}{K} \tag{4b}
\]

\[
\frac{\partial Y}{\partial U} = 1 - \frac{aL}{U} - \frac{bU}{K} \tag{4c}
\]

The requirement of non-negativity is equivalent to the following three inequalities:

\[
b > 0 \tag{5a}
\]

\[
aK > bU \tag{5b}
\]

\[
1 > \frac{aL}{U} + \frac{bU}{K} \tag{5c}
\]

The first condition (5a) is trivial. However the second and third conditions are not automatically satisfied for all possible values of the variables. It is therefore necessary to do the fitting by
constrained non-linear optimization. The statistical procedures and quality measures are discussed in Appendix B.

6. Results

![Graph showing LINEX production function fits with different “energy” factor inputs, USA 1900-1998](image)

The two curves in Figure 8 show the LINEX fit, with work $U_E$ and $U_B$ respectively as factors of production. In the first case, we consider physical work from commercial energy sources $U_E$ (excluding animal work) as a factor. The second case $U_B$, reflects work derived from all exergy inputs ($B$ includes animal work derived from agricultural phytomass.) The best fit, by far, is the latter. The unexplained residual has essentially disappeared, prior to 1975 and remains small thereafter. In short, it would seem that "technical progress" – as defined by the Solow residual – is almost entirely explained by historical improvements in exergy conversion (to physical work), as summarized in Figure 2, at least until recent times. The remaining unexplained residual, roughly 12% of recent economic growth (since 1975), is shown next in Figure 9.
We conjecture that a kind of phase-change or structural shift took place at that time, triggered perhaps by the so-called energy crisis, precipitated by the OPEC blockade. Higher energy prices induced significant investments in energy conservation and systems optimization. For instance, the CAFE standards for automobile fuel economy, introduced in the late 1970s, forced US motor vehicle manufacturers to redesign their vehicles. The result was to double the vehicle miles obtained from a unit of motor fuel in the US between 1970 and 1989. This was achieved mainly by weight reduction and improvements in aerodynamics and tires. Comparable improvements have been achieved in air travel, rail freight and in many manufacturing sectors, induced primarily by the sharp (though temporary) fuel price increases.

The marginal productivities of the factors can be calculated directly from equations (4).
The three marginal productivities for the preferred case, $U_B$, are plotted in Figure 10. The marginal productivity trends for capital and work, in both cases show a very slight directional change between 1970 and 1980. The marginal productivity of capital has started to increase whereas the marginal productivity of physical work – resulting from increases in the efficiency of energy conversion – has declined slightly. This shift roughly coincides with the two so-called oil crisis, and may well have been triggered by the spike in energy (exergy) prices that occurred at that time.

7. Summary and conclusions

In the `standard' model a forecast of GDP requires a forecast of labor $L$, capital stock $K$ and the Solow multiplier – multifactor productivity or technical progress — $A(t)$. We have shown that introducing energy and/or material resource (i.e. exergy) inputs does not significantly improve the explanatory power of traditional production functions. A time-dependent Solow-multiplier is still needed.

However a much better explanation of past economic growth can be obtained by introducing exergy services (useful work) as a factor of production, in place of exergy inputs. Exergy services can be equated to exergy inputs multiplied by an overall conversion efficiency.
which, of course, corresponds to cumulative technological improvements over time. Based on this hypothesis economic growth from 1900 to 1975 or so is explained almost perfectly, except for wartime perturbations. The results described above, the technical progress term can be decomposed into two main contributions. The most important, historically, is from improved exergy conversion-to-work efficiency. This propagates, via cost and price reductions, through the whole downstream value-added chain.

More recently, however, there is obviously some contribution from `other’ downstream technical improvements. Evidently growth of GDP in the past quarter century has slightly outstripped growth of the three main input factors, capital, labor and physical work. Since 1975 or so an additional source of value-added is involved. One possibility is energy conservation and systems optimization triggered by the energy (exergy) price spike in the 1973-1981 period. The other obvious candidate for this additional value creation is information and communications technology (ICT). However, in the spirit of some endogenous growth theories, it would be possible to interpret this additional productivity to some qualitative improvement in either capital or labor.

It does appear that the marginal productivity of physical work is still by far the dominant driver of past growth and will be for decades to come. This does not mean that human labor or capital are unimportant. As noted already, the three factors are not really independent of each other. Increasing exergy conversion efficiency requires investments of capital and labor, while the creation of capital is highly dependent on the productivity of physical work.

It is tempting to argue that the observed shift starting in the 1970s reflects the influence of information technology. Certainly large scale systems optimization depends very strongly on large data bases and information processing capability. The airline reservation systems now in use have achieved significant operational economies and productivity gains for airlines by increasing capacity utilization. Manufacturing firms have achieved comparable gains in machine utilization and inventory control through computerized integration of different functions.

One of the more important implications of the foregoing is that some of the most dramatic and visible technological changes of the past century have not contributed significantly to overall economic growth. An example in point is medical progress. While infant mortality has declined dramatically and life expectancy has increased very significantly since 1900, it is hard to see any direct impact on economic growth, at least up to the 1970s. Neither of the two major benefits adds to labor productivity. The gain has been primarily in quality of life, not quantity of output. Although health services demand an increasing share of GDP, there is no indication of a decline in prices, as implied by the positive feedback `growth engine’.

Changes in telecommunications technology since 1900 may constitute another example. New service industries, like moving pictures, radio and TV have been created, but if the net result is new forms of entertainment, the gains in employment and output may have come largely at the expense of earlier forms of public news and entertainment, such as the print media, live theater, circuses and vaudeville. Again, the net impact may have been primarily on quality of life. While the changes have been spectacular, as measured in terms of information transmitted, the productivity gains may not have been especially large, at least until recently (ethe 1990s) when the internet began to have an impact on ways of doing business.

In any case, since economic growth for the past century can be explained with considerable accuracy by three factors, $K, L, U$, it is not unreasonable to expect that future
growth for some time to come will be explained quite well by these same variables, plus a small but growing contribution from information and communications technology (ICT).

From a long-term sustainability viewpoint, this conclusion carries a powerful implication. If economic growth is to continue without proportional increases in fossil fuel consumption, it is vitally important to exploit new ways of generating value added without doing more work. But it is also essential to develop ways of reducing fossil fuel exergy inputs per unit of physical work output (i.e. increasing conversion efficiency). In other words, energy (exergy) conservation is probably the main key to long term environmental sustainability.

**Endnotes**

1. Research supported by institute for Advanced Study, UN University, Tokyo and The European Commission, TERRA project.

2. Marx believed (with some justification) that the gains would flow mainly to owners of capital rather than to workers. Political developments have changed the balance of power since Marx’s time. However, in either case, returns to energy or physical resources tend to decline as output grows. This can be interpreted as a declining real price.

3. For instance, for the years 1929 through 1969, one specification that gave good results without an exogenous term for technical progress was the choice of K and E as factors of production. In this case the best fit (R$^2 = 0.99895$) implied a capital share of only 0.031 and an energy share of 0.976, (which corresponds to very small increasing returns)[Hannon and Joyce 1981]. Another formulation, involving K and electricity, El, yielded very different results, namely (R$^2 = 0.99464$) a capital share of 0.990 with only a tiny share for electricity [ibid]. using factors K, L only — as Solow did in his pathbreaking (Nobel Prizewinning) paper — but not including an exogenous technical progress factor (as he did) the best fit (R$^2 = 0.99495$) was obtained with a capital share of 0.234 and a labor share of 0.852. These shares add up to more than unity (1.086), which implies significantly increasing returns. Evidently one cannot rely on econometrics to ascertain the “best” formulation of a Cobb-Douglas (or any other) production function.

4. There are statistical approaches to addressing the causality issue. For instance, Granger and others have developed statistical tests that can provide some clues as to which is cause and which is effect [Granger 1969; Sims 1972]. These tests have been applied to the present question (i.e whether energy consumption is a cause or an effect of economic growth) by Stern [Stern 1993; Kaufmann 1995]. In brief, the conclusions depend upon whether energy is measured in terms of heat value of all fuels (in which case the direction of causation is ambiguous) or whether the energy aggregate is adjusted to reflect the quality (or, more accurately, the price or productivity) of each fuel in the mix. In the latter case the econometric evidence seem to confirm the qualitative conclusion that energy (exergy) consumption is a cause of growth. Both results are consistent with the notion of mutual causation.

5. Georgescu-Roegen was the first to have emphasized this crucial point [Georgescu-Roegen 1966].

6. The US population in 1900 was 76 million, of which perhaps 50 million were of working age, but only 25 million were men (women worked, without question, but their work did not contribute to GDP at the time), and at least half of the male workers were doing things other than chopping wood, shoveling coal or lifting bales of cotton, which depended more on eye-hand coordination or intelligence than muscles. The minimum metabolic requirement is of the order of 1500 Cal/day (for men), whereas the average food consumption for a working man was about 3000 Cal/day, whence no more than 1500 Cal/day was available for physical work. This comes to 18 billion Cal/day or about 0.16 EJ/year of food exergy inputs for work.
compared to fossil fuel consumption of 8.9 eJ in that year. If muscles convert energy into work at about 15% efficiency, the overall food-to-work conversion efficiency for the human population as a whole would also be roughly 2.4%. In recent years, though most women have jobs, given the changing nature of work, and the much greater life expectancy and retirement time, the conversion efficiency has declined significantly.

7. In recent work subsequent to the submission of this paper, we have carried out a large number of statistical tests for both C-D and LINEX functions. The standard procedure is to carry out the OLS fit for annual increments, to eliminate possible collinearity. The results are essentially the same as those presented here.

References


Appendix A. Data

We have compiled a number of historical data sets for the US from 1900 through 1995, indexed to 1900. All of the series are from standard sources. Both labor and capital series up to 1970 are found in [USDOCEA 1973] *Long Term Economic Growth 1860-1970*, US Department of Commerce, Bureau of Economic Analysis. Tables (Series A 68 and A-65, respectively). More recent data (1947-1995) came from [USCEA 1996] *Economic Report of the President, 1996* (Tables B-32 and B-43). The earlier and later labor series are not exactly the same, but the differences during the period of overlap (1949-1970) are very minor. The capital series since 1929 comes from [USDOCEA] *Survey of Current Business*, May 1997, and [USDOC 1992] *Business Statistics*, also the US Department of Commerce. Labor is counted as man-hours actually worked, and private reproducible capital stock, adjusted by the fraction of the labor force actually employed. (This same adjustment was also made by Solow in his 1957 paper [Solow 1957].)

The exergy series are much more complicated. In brief, we have compiled historical data on fuel consumption for all fuels, including wood, and for non-fuel material inputs with non-trivial exergy content, including non-fuel wood, and major metal ores (iron, copper) and minerals (limestone). Data for 1900 to 1970 are mostly from [USCensus 1975] *Historical Statistics of the US from Colonial Times to 1970*, various tables, with some interpolations and estimates for missing numbers. More recent data on fuels – both raw and processed (including electricity) – are from [USDOEEIA annual] *Annual Review of Energy Statistics*. Data on other minerals and metal ores are from [USGS 1999; USBuMines annual] *Minerals Yearbooks* (US Bureau of Mines until 1995; US Geological Survey since then). We have calculated the exergy for all fuels as a multiplier of heat content; exergy for other materials was calculated using standard methods [Szargut et al 1988; Ayres et al 1998].

Finished materials include coal consumed by industry other than electric utilities, gas consumed by households or industry other than utilities, gasoline, heating oil, and residual oil (not consumed by utilities), plus electricity from all sources. Finished non-fuel materials with significant exergy content include plastics, petrochemicals, asphalt, metals, and non-fuel wood. Obviously large quantities of finished fuels are consumed by industry, for the manufacture of goods, and additional quantities are consumed in transporting those goods to final consumers (i.e. households).

There are no precise statistics on fuels and materials consumed by `final’ users *vis a vis* that which is consumed by intermediates. We do have a breakdown of energy usage since 1955, which distinguishes household use from industrial and commercial use. But transportation use is not subdivided in this way, either by the Department of Energy or the Department of Transportation. The best supplementary source for transportation energy use is Oak Ridge National Laboratory (ORNL). We have rather arbitrarily assigned all gasoline use to households and all diesel fuel use to commercial establishments. This undoubtedly overestimates household use, especially during the early decades of the century before small diesel engines became competitive. There is a further ambiguity, arising from the fact that as much as 40 percent of all automobile travel is for the purpose of travel to work. It could be argued that this fraction properly belongs to the `commercial’ category rather than the `private’ category, although we...
have not done so. Simply, we have calculated the household fraction of all fuels and assumed that the same percentage applies to the exergy content of all final goods.

The major time series for $K$, $L$, and $E$, expressed in index form normalized to 1900, are shown in Figure A-1. Conversion factors are given in Tables A-1 and A-2. Table A-3 (parts A, B, C, D) gives the original data sources.

References for this Appendix
Sources have often been abbreviated in the tables - these abbreviations are shown in bold capitals at the end of each relevant citation. Brackets indicate the main textual citation.
ANNERG = Annual energy review [USDOEEIA annual]
BUSTAT = Business statistics [USDOC 1992]
CEA = Economic Report of the President [USCEA 1996]
EIA = Annual energy review [USDOEEIA annual]
HNGA = Historical natural gas annual [USDOEEIA 1999]
IEA = Energy balances of OECD countries, annual [OECD/IEA 1999]
MINYB = Minerals yearbooks, annual [USGS 1999; USBuMines annual]
P&C = Potter and Christy [Potter and Christy 1968]
S&N = Schurr and Netschert [Schurr and Netschert 1960]
SCB = Survey of current business, monthly [USDOCBEA]
STATAB = Statistical abstract of the United States [STATAB annual]
Szar = Szargut [Szargut et al 1988]
### Table A-1: Conversion factors for petroleum products

<table>
<thead>
<tr>
<th>Substance</th>
<th>IBR</th>
<th>MBtu/bbl</th>
</tr>
</thead>
<tbody>
<tr>
<td>Crude oil</td>
<td>50</td>
<td>5.8</td>
</tr>
<tr>
<td>Asphalt/road oil</td>
<td>60.241</td>
<td>6.636</td>
</tr>
<tr>
<td>Distillate fuel oil</td>
<td>52.356</td>
<td>5.825</td>
</tr>
<tr>
<td>Jet fuel/kerosene</td>
<td>46.948</td>
<td>5.67</td>
</tr>
<tr>
<td>LPG</td>
<td>31.348</td>
<td>4.13</td>
</tr>
<tr>
<td>Motor gasoline</td>
<td>36.496</td>
<td>5.253</td>
</tr>
<tr>
<td>Residual fuel oil</td>
<td>55.866</td>
<td>6.287</td>
</tr>
<tr>
<td>Lubricants</td>
<td>52.083</td>
<td>6.065</td>
</tr>
<tr>
<td>Aviation gasoline</td>
<td>42.918</td>
<td>5.048</td>
</tr>
<tr>
<td>Other products</td>
<td>48.077</td>
<td>5.248</td>
</tr>
</tbody>
</table>
Table A-2: Typical chemical exergy content of some fuels

<table>
<thead>
<tr>
<th>Fuel</th>
<th>Exergy coefficient</th>
<th>Net heat. value</th>
<th>Chemical exergy</th>
</tr>
</thead>
<tbody>
<tr>
<td>Coal</td>
<td>1.088</td>
<td>21680 [KJ/kg]</td>
<td>23587.84 [KJ/kg]</td>
</tr>
<tr>
<td>Coke</td>
<td>1.06</td>
<td>28300 [KJ/kg]</td>
<td>29998 [KJ/kg]</td>
</tr>
<tr>
<td>Fuel oil</td>
<td>1.073</td>
<td>39500 [KJ/kg]</td>
<td>42383.5 [KJ/kg]</td>
</tr>
<tr>
<td>Natural gas</td>
<td>1.04</td>
<td>44000 [KJ/kg]</td>
<td>45760 [KJ/kg]</td>
</tr>
<tr>
<td>Diesel fuel</td>
<td>1.07</td>
<td>39500 [KJ/kg]</td>
<td>42265 [KJ/kg]</td>
</tr>
<tr>
<td>Fuelwood</td>
<td>1.15</td>
<td>15320 [KJ/kg]</td>
<td>17641 [KJ/kg]</td>
</tr>
</tbody>
</table>

_data source: expanded from SZAR_
### Table A-3a: Sources for coal data

<table>
<thead>
<tr>
<th>Material</th>
<th>Title</th>
<th>Period</th>
<th>Source</th>
<th>Mass (1 short ton = 0.9071847 metric tons)</th>
<th>Heat Content (1 Btu = 1055.056 joules)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>Reference</td>
<td>Series name and/or formula</td>
<td>Reference</td>
</tr>
<tr>
<td>Coal</td>
<td>Raw coal production</td>
<td>1949-1998</td>
<td>Annual Energy Review</td>
<td>Table 7.1, Col 1</td>
<td>Production</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Sum “Production”; Bituminous coal + Pennsylvania anthracite</td>
<td>Table A5 Col 1</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1850-1879</td>
<td>Historical Statistics - Volume 1</td>
<td>M93+M123</td>
<td>Sum “Production”; Bituminous coal + Pennsylvania anthracite</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Table 7.1, Col 6</td>
<td>“Coal consumption” = Production + Imports - Exports - Stock change - Losses &amp; unaccounted for</td>
</tr>
<tr>
<td>Coal</td>
<td>Raw coal apparent consumption</td>
<td>1949-1998</td>
<td>Annual Energy Review</td>
<td>Table 7.1, Col 6</td>
<td>“Coal consumption” = Production + Imports - Exports - Stock change - Losses &amp; unaccounted for</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Sum “Consumption in Btus”: Bituminous coal + Pennsylvania anthracite</td>
<td>M84+M85 interpolated before 1900</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1850-1879</td>
<td>Historical Statistics - Volume 1</td>
<td>M93+M123</td>
<td>Consumption assumed equal to production</td>
</tr>
<tr>
<td>Coal</td>
<td>Coal, apparent consumption as finished fuel</td>
<td>1949-1998</td>
<td>Annual Energy Review</td>
<td>Table 7.1, Col 6</td>
<td>“Coal consumption” = Production + Imports - Exports - Stock change - Losses &amp; unaccounted for</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Sum “Consumption in Btus”: Bituminous coal + Pennsylvania anthracite</td>
<td>M84+M85 interpolated before 1900</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Table 7.7, Col 5</td>
<td>Sum “Consumption in Btus”: Bituminous coal + Pennsylvania anthracite</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1890-1915</td>
<td>Historical Statistics - Volume 1</td>
<td>M85, M84, M114 extrapolated to zero in 1890, M122</td>
<td>“Coal consumption” = Production + Imports - Exports - Stock change - Losses &amp; unaccounted for</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Table 7.3, Cols 2, 8</td>
<td>Sum “Consumption in Btus”: Bituminous coal + Pennsylvania anthracite</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1872-1889</td>
<td>Historical Statistics - Volume 1</td>
<td>M85 , M84 interpolated, M122</td>
<td>“Coal consumption” = Production + Imports - Exports - Stock change - Losses &amp; unaccounted for</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Table 7.1, Col 6</td>
<td>Sum “Consumption in Btus”: Bituminous coal + Pennsylvania anthracite</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1850-1871</td>
<td>Historical Statistics V-1</td>
<td>M93+M123</td>
<td>Consumption assumed equal to production</td>
</tr>
</tbody>
</table>

**Note:** Multipliers (26.2, 25.4, 1.51) derived by exponential fits on years where both series were available.
### Table A-3b: Sources for petroleum data

<table>
<thead>
<tr>
<th>Material</th>
<th>Title</th>
<th>Period</th>
<th>Source</th>
<th>Metric ton: ( M(\text{product}) = F(\text{product})/B(\text{product}) )</th>
<th>Heat Content (1 Btu = 1055.056 joules)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Petroleum Energy = Heat ∗ 1.088</td>
<td>Crude oil production</td>
<td>1949-1998</td>
<td>Annual Energy Review Table 5.2, Col 8</td>
<td>( M(\text{crude oil production}) )</td>
<td>Table 1.2 Col 3: Production</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1859-1948</td>
<td>Schurr and Netschert Statistical Appendices Table A1:II, Col 4</td>
<td>( M(\text{crude oil production}) )</td>
<td>Table A1:II, col 4: Production</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1850-1858</td>
<td></td>
<td>zero</td>
<td></td>
</tr>
<tr>
<td>Petroleum products consumption as finished fuel</td>
<td>Crude oil apparent consumption</td>
<td>1949-1998</td>
<td>Annual Energy Review Table 5.2, Col 8 Table 5.1, Cols 5, 10</td>
<td>( M(\text{crude oil production} + \text{crude oil imports} - \text{crude oil losses}) ) with stock changes + net exports for crude oil per se assumed zero</td>
<td>Table 5.2, Col 8 Table 5.1, Cols 5, 10 times Table A2, Cols 1-2: ( M(\text{crude oil production} + \text{crude oil imports} - \text{crude oil losses}) ) with stock changes + net exports for crude oil per se assumed zero</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1859-1948</td>
<td>Schurr and Netschert Statistical Appendices Table A1:VI, Col 4</td>
<td>( M(\text{crude oil apparent consumption}) )</td>
<td>Table A1:VII, Col 4: Apparent crude oil consumption</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1850-1858</td>
<td></td>
<td>zero</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Petroleum products consumption as finished fuel</td>
<td>1949-1998</td>
<td>Annual Energy Review (EIA) Table 5.12a, Cols 1-5, 7-14 Table 5.12b, Cols 1, 7</td>
<td>Finished fuel = ( M(\text{Asphalt/road}) + M(\text{Distillate}) + M(\text{Jet}) + M(\text{LPG total}) + M(\text{Gasoline}) + M(\text{Residual}) + M(\text{Other}) ) for residential/commercial, industrial &amp; transport</td>
<td>Table 2.1, cols 3, 9, 13: Finished fuel = consumption by residential, commercial, industrial and transport</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1920-1948</td>
<td>Schurr and Netschert Historical Statistics Vol II Table A1:VI, Col 4 Table 8.8 Col 5 (EIA) Table ILS45 (HIST)</td>
<td>Finished fuel = Apparent consumption (A1V14) - Energy sector use (8.85 extrapolated to zero in 1876 using rates from ILS45)</td>
<td>Table A1:VII Col 4 Table 8.8 Col 6 (EIA) Table ILS45 (HIST): Finished fuel = Apparent consumption (A1V14) - Energy sector use (8.85 extrapolated to zero in 1876 using rates from ILS45)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1850-1858</td>
<td></td>
<td>zero</td>
<td></td>
</tr>
</tbody>
</table>

Note on finished fuel calculation: Comparison of values in Annual Energy Review from Table 5.12b (energy sector use) and Table 8.8 (electric utility use) in common units produce similar numbers for 1949-1998. This suggests that internal use by the petroleum industry of petroleum products has been excluded from apparent consumption. Hence it has not been subtracted twice.
<table>
<thead>
<tr>
<th>Material</th>
<th>Title</th>
<th>Period</th>
<th>Source</th>
<th>Mass (cubic feet=metric tons*50875.05)</th>
<th>Heat Content (1 Btu = 1055.056 joules)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Natural gas</td>
<td>Historical Natural Gas Annual</td>
<td>1936-1998</td>
<td>Table 1, Col 1</td>
<td>Gross withdrawals</td>
<td>Table 1, Col 1, EIA. A4, Col 1</td>
</tr>
<tr>
<td>Base units = million cubic feet</td>
<td></td>
<td></td>
<td></td>
<td>Gross withdrawals(7.1)* Dry production factor(A4.1)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Historical Natural Gas Annual</td>
<td>1930-1935</td>
<td>Table 1, Col 5</td>
<td>1.25<em>marketed production (1.25</em>T1.5)</td>
<td>Table 1, Col 5, EIA.A4, Col 1</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>1.25<em>marketed production</em> Dry production factor(A4.1)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Schurr &amp; Netschert Statistical Appendix I</td>
<td>1882-1929</td>
<td>Table I, Col 5</td>
<td>1.25<em>marketed production (1.25</em>T1.5)</td>
<td>Constant 1.035 from EIA.A4</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>1.035<em>1.25</em>marketed production*</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Schurr &amp; Netschert Statistical Appendix I</td>
<td>1850-1881</td>
<td>zero</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Natural gas apparent consumption</td>
<td>Historical Natural Gas Annual</td>
<td>1930-1998</td>
<td>Table 2, col 8, Table 1, col 6</td>
<td>Consumption (T2.8) + NGL (T1.6)</td>
<td>Table 2, Col 8, Table 1, col 6, Table A4, Cols 1, 2</td>
</tr>
<tr>
<td>includes natural gas liquids</td>
<td></td>
<td></td>
<td></td>
<td>Dry consumption (T2.8<em>A4.1) + NGL (T1.6</em>A4.2)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Schurr &amp; Netschert Statistical Appendix I</td>
<td>1882-1930</td>
<td>Table VI, Cols 5 &amp; 6</td>
<td>Consumption (natural gas + NGL) interpolated 1882-1890</td>
<td>Table VII, Cols 5 &amp; 6 Statistical Appendix I</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Consumption (natural gas + NGL) interpolated 1882-1890</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Schurr &amp; Netschert Statistical Appendix I</td>
<td>1850-1881</td>
<td>zero</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Natural gas consumption as finished fuel (excludes NGL)</td>
<td>Historical Natural Gas Annual</td>
<td>1930-1998</td>
<td>Table 3, Col 8 Table 3, Col 7</td>
<td>Finished fuel = Total delivered to consumers (T3.8) - electric utility use(T3.7) (total delivers includes pipeline and plant use). Same as sum/residential, commercial, industrial &amp; transport (T3.1+T3.4+T3.5+T3.6)</td>
<td>Table 3, Col 8 - Table 3, Col 7 EIA A4, Cols 3 &amp; 4</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Delivered to consumers (T3.8<em>A4.4) - electric utility use (T3.7</em>A4.3)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Schurr &amp; Netschert Statistical Appendix I Historical Natural Gas Annual</td>
<td>1890-1929</td>
<td>Table 3, Cols 8,7 extrapolated to zero in 1882 using rates from S&amp;N Table VI, Cols 5, 6</td>
<td>Finished fuel = Delivered to consumers (T3.8 via V1.6) - electric utility use (T3.7 via V1.7)</td>
<td>Table 3, Cols 8,7 extrapolated Constant factor 1.035</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Finished fuel = 1.035 * [Delivered to consumers (T3.8 via V1.6) - electric utility use (T3.7 via V1.7)]</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Schurr &amp; Netschert Statistical Appendix I Historical Natural Gas Annual</td>
<td>1850-1881</td>
<td>zero</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Note: The multiplier 1.25 (marketed for gross) derived from fit on years where both series were available. The constant 1.035 is inferred from all values prior to 1940 in Table A4 of the Natural Gas Annual.
### Table A-3d: Sources for fuelwood and biomass data

<table>
<thead>
<tr>
<th>Material</th>
<th>Title</th>
<th>Mass (million cubic feet roundwood equivalent*(0.017 to 0.022)=MMT. Multiplier time dependent)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td><strong>Period</strong></td>
</tr>
<tr>
<td></td>
<td>Fuel wood</td>
<td>1997-1998</td>
</tr>
<tr>
<td></td>
<td>Exergy= Heat*1.152</td>
<td>1965-1996</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1958-1964</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1900-1957</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1850-1899</td>
</tr>
</tbody>
</table>

#### Heat Content (1 Btu = 1055.056 joules)

<table>
<thead>
<tr>
<th>Period</th>
<th>Source</th>
<th>Reference</th>
<th>formula</th>
</tr>
</thead>
<tbody>
<tr>
<td>1981-1998</td>
<td>Annual Energy Review</td>
<td>Table 10.3, Row 1</td>
<td>Wood energy</td>
</tr>
<tr>
<td>1970-1980</td>
<td></td>
<td>Table 10.3 row 1 &amp; Table 1.2, Col 10</td>
<td>Wood energy &amp; Energy from biomass, adjusted and interpolated</td>
</tr>
<tr>
<td>1949-1969</td>
<td></td>
<td>Table 1.2, Col 10</td>
<td>Energy from biomass (=fuelwood only)</td>
</tr>
<tr>
<td>1850-1949</td>
<td>Historical Statistics V.1</td>
<td>M92, interpolated</td>
<td>Fuel wood consumption</td>
</tr>
</tbody>
</table>


Appendix B

The LINEX function parameters were obtained by using a quasi-Newton non-linear optimization method, with box-constraints. The objective function was simply the sum of the squared error. The constraints on the possible values of the parameters of the LINEX model were required to ensure that the factor marginal productivities (equations 4a-4c) were non-negative. A statistical measure of the overall fit was provided by the mean square error

\[ \text{MSE} = \frac{\sum_{t=1}^{n} e_t^2}{n-k} \]  

(B-1)

The MSE is a measure of the absolute deviation of the theoretical fit from the empirical curve, where \( n \) is the number of samples, \( k \) the number of parameters and \( e \) the residual from the fitted curve.

To compare the predictive power of raw exergy flows (\( B \) and \( E \)) \textit{vis a vis} physical work (\( U_B \) and \( U_B \)) we calculated the correlation coefficients (\( R^2 \)) between the logarithms of the LINEX function of the variables and the actual GDP. We tested the significance of the correlations using a t-test with Welch modification for unequal variances (Table B-1). The results showed that the relationship of GDP with a LINEX function of \( B \) or \( E \) was not significant. Substituting \( U_E \) and \( U_B \) the correlations were significant, and the latter choice was by far the most significant, as indicated by the small t-value.

<table>
<thead>
<tr>
<th>Variable used</th>
<th>t-value</th>
<th>Degrees of freedom</th>
<th>p-value</th>
<th>Correlation Coefficient (( R^2 ))</th>
</tr>
</thead>
<tbody>
<tr>
<td>( B )</td>
<td>5.74</td>
<td>158.4</td>
<td>4.5e-08</td>
<td>0.98</td>
</tr>
<tr>
<td>( E )</td>
<td>3.09</td>
<td>174.3</td>
<td>0.002</td>
<td>0.97</td>
</tr>
<tr>
<td>( U_E )</td>
<td>0.51</td>
<td>194.8</td>
<td>0.604</td>
<td>0.99</td>
</tr>
<tr>
<td>( U_B )</td>
<td>0.19</td>
<td>195.9</td>
<td>0.845</td>
<td>0.99</td>
</tr>
</tbody>
</table>

The coefficient of determination is often reported as another measure of the goodness of fit. To be valid the residuals should be identically and independently distributed. The Durbin-Watson statistic was used to check for the presence of correlated residual error. It is calculated as

\[ DW = \frac{\sum_{t=2}^{n} (e_t - e_{t-1})^2}{\sum_{t=1}^{n} e_t^2} \]  

(B-2)

where \( e \) is the residual error, calculated for each year \( t \), for a time period of length \( n \), where \( k \) is the number of independent variables. The \( DW \) statistic takes values between 0 and 4. The
Null Hypothesis was that there was no significant correlation between the residual error values for each year.

The DW statistic, calculated for each estimate using either definition of work input ($U_E$ or $U_B$) for the entire period (1900-1998), provided evidence of strong residual autocorrelation ($DW < 1.61$, $k=3$). The Null Hypothesis was therefore rejected. However for the estimate made using $U_B$, the $DW$ statistic for the period 1900-1975 was 0.2387. The Null Hypothesis was accepted for this shorter time period because the post 1975 residual was excluded from the analysis. Interestingly this same result was not found for the fit (1900-1975) using $U_E$. 